DETERMINATION OF THE FIRST CRITICAL THERMAL FLUX ON FLAT HEATERS

Yu. A. Kirichenko and P. S. Chernyakov

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A relation is derived for the first critical thermal flux as a function of the contact angle when a large volume of liquid boils at a flat heater surface. Theoretical results are compared with experimental ones.

A hydrodynamic theory has been developed in [1-7] concerning the critical stages of boiling in a large volume. The hypothesis about the hydrodynamic character of the critical stages was first stated by S. S. Kutateladze in [1], where the processing of experimental data by the methods of similarity theory yielded the following relation:

$$\frac{q_{cr}}{L\rho_2}\left(\frac{\rho_1}{g\sigma}\right)^{0.25} = 0.17\left(\frac{\rho_1}{\rho_2}\right)^{0.48}.$$

The hydrodynamic theory, which involves the theory of hydrodynamic stability during the critical stages in boiling, has been further generalized in [2-4] and, as a result, the following expression was obtained for q_{cr}:

$$\frac{q_{cr}}{L\sqrt{\rho_2}\left(g\sigma(\rho_1-\rho_2)\right)^{0.25}} = \varphi = \text{const.} \tag{1}$$

It has been shown in [2] that φ is a function of pressure and varies from 0.13 to 0.2. Much work has been done to obtain a more accurate value for the constant φ and to determine how it depends on the physical properties of the liquid as well as on the modal parameters of the boiling process; in some articles the expression for q_{cr} is derived analytically in a form approaching that of expression (1) [5-13]. On the basis of expression (1) and using the theory of thermodynamic similarity, V. M. Borishanskii has obtained a relation for φ as a function of pressure [2, 5]. A variant of expression (1), containing instead of φ a quantity slightly varying with viscosity, was obtained in [6]. The first critical boiling stage is seen here as resulting from a disturbed hydrodynamic equilibrium of the liquid jets surrounded by vapor streams.

For the first time q_{cr} was determined analytically in [2, 7] and it was determined there accurately, except for a constant based on experimental data. According to the procedure proposed in [6, 7], N. Zuber has established q_{cr} for the case of potential flow [8]. Articles [9, 10] further develop the ideas of article [8].

The dependence of the critical thermal flux on the viscosity of the liquid has been derived in [11, 12, 13] with the assumption that the diameters of the liquid jets are proportional to the bubble diameter before separation as well as to the ratio of areas occupied by the liquid and the vapor on the heater surface. The expression for q_{cr} given in [11, 12, 13] contains three constants which are determined by test. Certain experimental results [14] indicate a rather strong dependence of the first critical thermal flux on the contact angle, but there is no theoretical interpretation for this to be found in articles on the critical boiling stages [1-13]. We will subsequently explain this relation between q_{cr} and contact angle θ. The present article is an extension of the work done in [2, 6-13].

We consider the boiling of a liquid at the surface of a heater located inside an infinitely large volume of this liquid.

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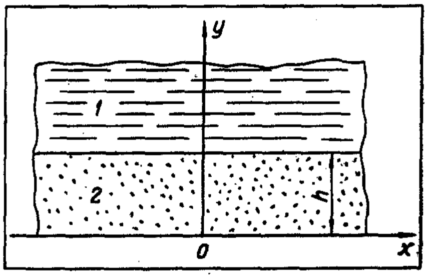

Fig. 1. Schematic diagram for the calculations: 1) liquid, 2) vapor.

We assume that 1) the liquid flow and the vapor flow are potential flows, 2) the liquid and the vapor are incompressible media, 3) the process occurs with continuous vapor columns. We will solve the given problem starting out with a boiling pattern similar to that shown in [2, 6, 7]. At thermal flux densities near q_{cr}, vapor columns rise from the heater surrounded by descending liquid jets. At a thermal flux density equal to q_{cr}, the vapor columns merge and an unstable vapor film forms on the heater surface. This is brought about by a loss of stability in the two-phase flow. In order to determine q_{cr}, it is necessary to calculate the critical diameters of vapor columns and of liquid jets along with the critical velocities of both the liquid and the vapor. The critical diameters will be found considering that they are determined by the instability of the horizontal liquid–vapor interface which is formed when vapor bubbles merge at the heater surface, i.e., they are determined by the Taylor instability [8, 9, 10]. Let us consider the stability of the liquid–vapor interphase boundary according to Taylor. The flat heater is assumed to be covered with a vapor film whose thickness is equal to the breakaway diameter of the bubbles and which is surrounded by an infinitely large volume of liquid (Fig. 1). The breakaway diameter of bubbles will be calculated by the Fritz formula [4]:

$$h = 0.018\theta \sqrt{\frac{\sigma}{(\rho_1 - \rho_2) g}}. \tag{2}$$

Let φ_1 and φ_2 be the velocity potentials of the perturbed liquid and vapor flows. The equations for φ_1 and φ_2 are

$$\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2} = 0, \quad \frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2} = 0. \tag{3}$$

The boundary conditions are

$$\left.\frac{\partial \varphi_2}{\partial y}\right|_{y=0} = 0, \quad \varphi_1|_{y=\infty} = 0. \tag{4}$$

The conditions at the interphase boundary y = h are

$$p_2 = p_1 - \frac{\sigma}{h^2}\left(\xi + h^2 \frac{\partial^2 \xi}{\partial x^2}\right), \tag{5}$$

$$p_1 = \rho_1\left(\frac{\partial \varphi_1}{\partial t} - g\xi\right), \tag{6}$$

$$p_2 = \rho_2\left(\frac{\partial \varphi_2}{\partial t} - g\xi\right), \tag{7}$$

$$\frac{\partial \xi}{\partial t} = -\frac{\partial \varphi_1}{\partial y} = -\frac{\partial \varphi_2}{\partial y}. \tag{8}$$

We seek

$$\varphi_1 = C \exp(\omega t + ikx - ky),$$
$$\varphi_2 = \exp(\omega t + ikx)\bigl(A \exp(ky) + B \exp(-ky)\bigr). \tag{9}$$

Inserting (9) into (4) and (8), we have

$$\varphi_1 = A\bigl(1 - \exp(2kh)\bigr) \exp(\omega t + ikx - ky),$$
$$\varphi_2 = 2A \exp(\omega t + ikx) \operatorname{ch} ky,$$
$$\xi = -\frac{2Ak}{\omega} \exp(\omega t + ikx) \operatorname{sh} kh. \tag{10}$$

Having inserted (10) into (6) and (7), we find

$$p_1 = \frac{2\rho_1 A}{\omega} \exp(\omega t + ikx) \operatorname{sh} kh (\omega^2 + gk),$$

TABLE 1

θ^0	10	20	30	40	50	60	70	80
φ	0,406	0,294	0,248	0,224	0,209	0,199	0,193	0,189

$$p_2 = \frac{2\rho_2 A}{\omega} \exp(\omega t + ikx)\,[\omega^2 \operatorname{ch} kh + gk \operatorname{sh} kh]. \tag{11}$$

Inserting next (11) and (10) into (5) yields

$$\rho_1 \operatorname{sh} kh\,(\omega^2 + gk) - \rho_2\,(\omega^2 \operatorname{ch} kh + gk \operatorname{sh} kh) = \frac{\sigma k}{h^2} \operatorname{sh} kh\,(h^2k^2 - 1).$$

Using the principle of stability change [15] and also expression (2), we find k:

$$k = \sqrt{\frac{(\rho_1 - \rho_2)\,g}{\sigma}} \cdot \frac{\sqrt{1 + 0.324 \cdot 10^{-3}\theta^2}}{0.018\theta}. \tag{12}$$

The wavelength of perturbations λ is determined by the formula

$$\lambda = 2\pi/k.$$

It follows from here that the liquid–vapor interphase boundary tends to expand into square area cells with the sides equal to λ. Let it be assumed that at each such element with an area equal to λ^2 there appears a liquid jet whose diameter is equal to a [9] according to the formula

$$a = \frac{\lambda}{2} = \frac{\pi}{k}. \tag{13}$$

The dimension thus chosen is somewhat arbitrary but within the correct order of magnitude. It follows from (13), indeed, that the areas ratio (A_1 occupied by vapor and A_2 occupied by liquid) is equal to 4.092. From the law of conservation of matter we have:

$$\frac{A_1}{A_2} = \frac{x_0}{1 - x_0}.$$

It has been shown in [16, 17] that $x_0 \geq 0.8$; in our case the areas ratio corresponds to an 80.3% volume content of vapor in the wall layer. Thus, the magnitude of a liquid jet diameter determined by the formula in [16] agrees with experimental data. Therefore, the critical diameters of liquid jets descending toward the heater can be calculated by the formula:

$$a = \frac{0.018\pi\theta}{\sqrt{1 + 0.324 \cdot 10^{-3}\theta^2}} \sqrt{\frac{\sigma}{(\rho_1 - \rho_2)\,g}}. \tag{14}$$

We will now determine the critical velocity of the vapor. A precise determination of this velocity can be made by considering the stability of the composite systems which consists of vapor columns (diameter d) ascending at a velocity U" and water jets (effective diameter a) descending at a velocity U. With the relation between U, U", d, and a, which has been derived by analyzing the stability of such a system, and with the law of continuity as well as with formulas (12), (13), (14) we can calculate the magnitude of the first critical thermal flux. For this purpose, as has been done in [6-13], we will solve the following simplified problem: let a cylindrical liquid jet descend toward the heater while the vapor inside this jet flows at a velocity U". The length of this cylindrical jet is assumed to be infinite. In order to find the critical velocity of the vapor, we make use of the solution to the problem concerning the stability of a cylindrical jet [18]. The following equation has been obtained in [18] for the increment of interphase boundary fluctuations:

$$\alpha^2 = \frac{\sigma k}{\rho_1 L_0^2}(1 - k^2 L_0^2)\,\frac{I_1(kL_0)}{I_0(kL_0)} + \frac{\rho_2 k^2 U''^2}{\rho_1}\,\frac{K_0(kL_0)}{K_1(kL_0)} \cdot \frac{I_1(kL_0)}{I_0(kL_0)}, \tag{15}$$

where $L_0 = 0.5a$.

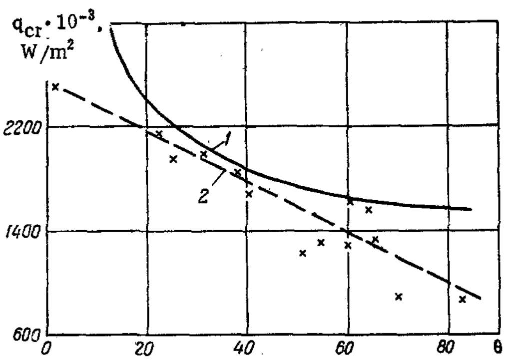

Fig. 2. First critical thermal flux $q_{cr} \cdot 10^{-3}$ (W/m^2) at flat heater surfaces, as a function of the contact angle: 1) theoretical relation, 2) experimental relation (dots indicate test points).

Let the wavelength of the interphase boundary fluctuations be λ. Then, expression (13) and (15) together with the principle of stability change [15] will yield an expression for the critical vapor velocity U'':

$$U'' = \sqrt{\frac{\sigma k d_2}{\rho_2}}, \tag{16}$$

where $d_2 \approx 0.77$.

The magnitude of the first critical thermal flux [9] becomes

$$q_{cr} = \frac{\pi}{16} L \rho_2 U''. \tag{17}$$

Inserting (13), (14), and (16) into (17) gives

$$\varphi = \frac{q_{cr}}{L\sqrt{\rho_2}\sqrt[4]{\sigma(\rho_1 - \rho_2) g}} = 0.171 \frac{\sqrt[4]{1 + 0.324 \cdot 10^{-3}\theta^2}}{\sqrt{0.018\theta}}. \tag{18}$$

The following values of the first critical thermal flux φ, in dimensionless units, as a function of the contact angle θ have been calculated by formula (18).

With the aid of this relation between φ and θ we can determine the first critical thermal flux for water and we will compare the calculated results with the test results obtained in the experiment [14] (Fig. 2). It is evident from Fig. 2 that the results of calculations and tests are in close agreement for water within the practically important range of contact angles $20° \le \theta \le 60°$. One must note that formula (18) has certain limitations. It is obviously not applicable to contact angles close to 0°, since, according to it, $\lim_{\theta \to 0} q_{cr} = \infty$. This is connected with the fact that the Fritz formula does not apply to the breakaway diameter of a bubble in the case of small contact angles. Furthermore, the numerical factor 0.171 on the right-hand side of the equality is somewhat too high. Relation (18) and, particularly, the numerical factor can be determined more accurately with a sufficient amount of experimental data on the breakaway diameter of vapor bubbles and on the critical thermal flux as a function of the contact angle.

NOTATION

x, y	are the Cartesian coordinates;
h	is the thickness of vapor film;
p_1, p_2	are the liquid and vapor pressures;
φ_1, φ_2	are the liquid and vapor velocity potentials;
t	is the time;
ξ	is the excursion from the free equilibrium surface in the normal direction;
g	is the gravity acceleration;
k	is the wave number;
ω	is the frequency;
ρ_1, ρ_2	are the liquid and vapor densities;
σ	is the surface tension;
θ	is the contact angle;
λ	is the wavelength;

a	is the diameter of liquid jet;
d	is the diameter of vapor column;
A_1	is the area occupied by vapor;
A_2	is the area occupied by liquid;
x_0	is the volume content of vapor in the wall layer;
$L_0 = a/2$;	
U, U"	are the critical velocities of liquid and vapor respectively;
α	is the increment of fluctuations;
q_{cr}	is the critical thermal flux;
L	is the latent heat of evaporation;
$\varphi = q_{cr}/L\sqrt{\rho_2}\sqrt[4]{\sigma(\rho_1-\rho_2)g}$.	

LITERATURE CITED

1. S. S. Kutateladze, Heat Transfer during Condensation and Boiling [in Russian], Mashgiz, (1949).
2. V. M. Borishanskii, Analysis of Transient Boiling Modes [in Russian], Cand. dissert., TsKTI, Leningrad (1950).
3. S. S. Kutateladze, Zh. Tekh. Fiz., 20, No. 11, 1389 (1950).
4. S. S. Kutateladze, Fundamentals of Heat Transfer Theory [in Russian], Mashgiz (1962).
5. V. M. Borishanskii, Kotloturbostroenie, No. 4, 22 (1952).
6. V. M. Borishanskii, Zh. Tekh. Fiz., 26, No. 2, 452 (1956).
7. V. M. Borishanskii, Trudy TsKTI, No. 28 (1955).
8. N. Zuber, Trans. ASME, J. Heat Transfer, 80, No. 11 (1958).
9. L. Leppert and K. Pitts, in: Heat Transfer Problems [Russian translation], Atomizdat (1967).
10. R. Moissis and P. Berenson, Teploperedacha, 83, No. 3, 39 (1963).
11. V. M. Borishanskii and M. A. Gotovskii, Trudy TsKTI, No. 58, 52 (1961).
12. V. M. Borishanskii and M. A. Gotovskii, Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1965).
13. V. M. Borishanskii and M. A. Gotovskii, in: Heat and Mass Transfer [in Russian], Moscow (1965), 3rd ed., p. 125.
14. V. I. Yashnov, Trudy TsKTI, No. 58, 78 (1965).
15. Lin-Tsia-Tsiao, Theory of Hydrodynamic Stability [Russian translation], Izd. IL (1958).
16. I. G. Malenkov, Zh. Prikl. Mekhan. i Tekh. Fiz., No. 6 (1963).
17. M. A. Styrikovich and E. I. Nevstrueva, Teplofiz. Vys. Temper., 2, No. 3 (1964).
18. V. G. Levich, Physico-Chemical Hydrodynamics [in Russian], Fizmatgiz (1959).